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High intrinsic noise and absence of hysteresis in superconducting quantum interference devices with large Steward-McCumber parameter

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We investigated niobium thin film superconducting quantum interference devices (SQUIDs) with large Steward-McCumber parameter ($\beta_c > 1$). No hysteresis was observed in the current-voltage (I-V) characteristics of the SQUIDs, even for $\beta_c \approx 17$. We attribute the absence of hysteresis to an excess voltage noise of the junctions which increases the SQUID intrinsic noise $\delta\Phi_s$. It can be represented by an effective noise temperature T^* of the SQUID which is higher than the bath temperature T. We simulated SQUID I-V characteristics using the measured device parameters and confirmed the absence of hysteresis. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4816730]

There are two main noise sources in any direct-current (dc) superconducting quantum interference device (SQUID) system, the SQUID intrinsic noise $\delta\Phi_s$ and the noise $\delta\Phi_{preamp}$ produced by the preamplifier of the SQUID readout circuit. In order to obtain a low intrinsic noise, one usually designs the SQUID with a small McCumber parameter $\beta_c = 2\pi I_c C R_J^2/\Phi_0 \leq 1$,[1] even $\leq 0.3 \sim 0.4$.[2] Here, I_c denotes the junction's critical current, C its self-capacitance, R_J the junction shunt resistance, and Φ_0 the magnetic flux quantum. In a conventional SQUID system, a readout electronics with flux modulation scheme (FMS)[3] is employed to suppress the preamplifier noise. Without flux modulation, additional positive feedback circuitries can suppress $\delta\Phi_{preamp}$.[4–6] Indeed, reading out the SQUID by a second SQUID usually yields the lowest preamplifier noise.[7]

The behavior of current-biased weakly damped junctions (i.e., $\beta_c > 1$) had been studied in several publications. One can infer from simulations by Tesche and Clarke[8] that the value of β_c at which hysteresis appears in the quasi-static I-V characteristics is greater than one in the case of large thermal noise, while in the case of negligible noise, hysteresis appears at $\beta_c \approx 1$. For a quantitative description, they introduced the thermal rounding parameter $\Gamma = 2\pi k_B T/I_c\Phi_0$, where k_B denotes the Boltzmann constant, and T denotes the temperature. Tunnel junction simulations by Voss pointed out that the hysteresis can disappear for $\beta_c \geq 1$ while an excess voltage noise S_v appears across the junctions,[9] thus rounding the I-V characteristics and preventing a hysteresis. In SQUID operation, although S_v will increase the SQUID intrinsic noise $\delta\Phi_s$,[10] SQUIDs with large β_c exhibit a large voltage-to-flux transfer coefficient $\partial V/\partial\Phi$, which can conveniently be used to reduce $\delta\Phi_{preamp}$.[11–13] In the so-called direct readout scheme (DRS), the SQUID can be then directly connected to an operational amplifier without the need for feedback circuitries to enhance $\partial V/\partial\Phi$, which simplifies the SQUID system substantially.

In this work, we measured a large number of SQUIDs in the current bias mode and determined their white noise, $\delta\Phi_s$, the flux-to-voltage transfer coefficient, $\partial V/\partial\Phi$, the critical current of the junctions, I_c, and the dynamic resistance at the working point, R_d, all at different values of β_c. Up to β_c approaching 20, we did not observe hysteresis but noted an increase in the intrinsic SQUID noise with β_c. The effective noise temperature $T^* > T_{bath} = 4.2$ K leads to an effective thermal rounding parameter Γ^*, which explains the absence of hysteresis.

In our experiments, we employed planar niobium-SQUID magnetometers with a pick-up-loop size of 5 $\times$ 5 mm^2 and a SQUID inductance of $L_s = 350$ pH.[11–13] The field-to-flux transfer coefficient $\partial B/\partial\Phi$ of these magnetometers reached 1.5 nT/Φ_0, approximately. We used two readout schemes to measure SQUID parameters and noise in flux-locked-loop (FLL): the traditional FMS using a step-up transformer, which reduces the effective voltage noise of the preamplifier to 0.08 nV/$\sqrt{}$Hz (at 100 kHz) when the primary coil of the transformer is shorted, and a simple DRS, without any feedback to enhance $\partial V/\partial\Phi$, i.e., the SQUID was directly connected to an operational amplifier used as a preamplifier (in our case AD 797). All SQUID chips were placed in a niobium shielding tube to perform the noise measurements.

It is difficult to determine β_c, which is a product of I_c, C, and R_J^2 experimentally, because the effective values of C and R_J cannot be measured easily for the high-frequency Josephson currents. We obtained a nominal β_c by using the measured value for I_c, the design value of the junction normal resistance, R_J (or R_J measured from I-V characteristics at large bias current), and a value of $C = 360$ fF, estimated from the junction's geometry. Generally, this capacitance

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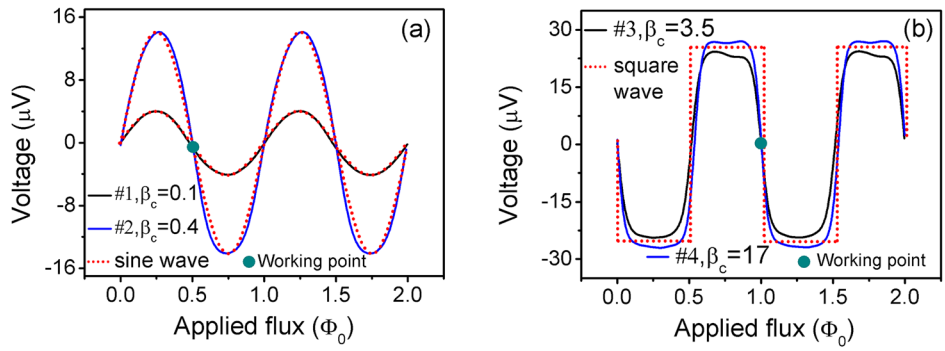

FIG. 1. Measured V-Φ curves of current-biased SQUIDs: (a) in the case of low nominal β_c of 0.1 (SQUID #1, $R_J = 5\ \Omega$) and 0.4 (#2, $R_J = 10\ \Omega$), and (b) in the case of high β_c of 3.5 (#3, $R_J = 30\ \Omega$) and 17 (#4, $R_J = 65\ \Omega$). Flux scale origin is arbitrary. The measured $2I_c$ of all SQUIDs was around 8 μA. For $\beta_c < 1$, the shape of the V-Φ curves is quasi-sinusoidal but quasi-rectangular for $\beta_c > 1$, with no hysteresis observed. Sine and a square waves are traced with dotted lines to guide the eye.

consists of two parts: the junctions and the niobium traces of the counter electrode overlaying the SQUID washer. However, this capacitance may be influenced by the parasitic capacitances in the layout structures.[14] To obtain a wide range of β_c values for a given SQUID geometry, R_J was varied while keeping constant I_c and C.

For meaningful statistics, we measured about 300 SQUID magnetometers with different β_c values. We used a DRS with a wide bandwidth (>1 MHz) to record the SQUID V-Φ and I-V curves and to derive $\partial V/\partial\Phi$ and R_d values. As β_c changes from $\beta_c \ll 1$ to larger values, the V-Φ curves change from a sine wave to a square wave, the latter indicating a $\beta_c \gg 1$. The measured V-Φ curves of SQUIDs with four typical values of β_c are shown in Figure 1.

The V-Φ curves shown for $\beta_c < 1$ (SQUID #1 and #2), are quasi-sinusoidal. With a β_c value increasing from 0.1 to 0.4, the voltage swing V_{swing}, the flux-to-voltage transfer coefficient $\partial V/\partial\Phi$, and R_d at working point obtained from the I-V characteristics, increased from 8 μV to 28 μV, 30 μV/Φ_0 to 110 μV/Φ_0, and 3.5 Ω to 10 Ω, respectively.

The V-Φ curves of the two SQUIDs with $\beta_c > 1$ (SQUID #3 and #4), shown in Figure 1(b), are nearly rectangular without any hysteresis. In this range of β_c, V_{swing} increased only slightly, from 50 μV ($\beta_c = 3.5$) to 54 μV ($\beta_c = 17$) while both $\partial V/\partial\Phi$ and R_d increased further from 350 μV/Φ_0 to 600 μV/Φ_0 and 25 Ω to 50 Ω. All SQUID parameters measured by DRS are listed in Table I, including the noise analysis.

TABLE I. SQUID parameters measured by DRS.

SQUID No.		#1	#2	#3	#4
β_c[a]	…	0.1	0.4	3.5	17
R_J[b]	Ω	5	10	30	65
R_d[c]	Ω	3.5	10	25	50
V_{swing}[d]	μV	8	28	50	54
$\partial V\partial\Phi$[e]	$\mu V/\Phi_0$	30	110	350	600
$\delta\Phi_{preamp}$	$\mu\Phi_0/\sqrt{Hz}$	33	9.1	2.9	1.7
$\delta\Phi$[f]	$\mu\Phi_0/\sqrt{Hz}$	33	9.3	4.2	7.2
$\delta\Phi_s$	$\mu\Phi_0/\sqrt{Hz}$	…[g]	1.9	3.0	7

[a,b]The nominal value.
[c]Derived from the I-V characteristics.
[d,e]Obtained from traces of Figure 1.
[f]Measured SQUID system noise in FLL mode.
[g]Not determined.

To verify that the measured system flux noise $\delta\Phi$ of SQUID #4 is dominated by $\delta\Phi_s$, the noise spectrum was also measured by FMS readout electronics. Noise levels of this SQUID were almost the same as shown in Figure 2.

The measured system noise $\delta\Phi$ listed in Table I is the sum of $\delta\Phi_s$ and $\delta\Phi_{preamp}$, i.e., $\delta\Phi = [\delta\Phi_s{}^2 + \delta\Phi_{preamp}{}^2]^{1/2}$. In our DRS, a commercial operational amplifier with a voltage noise $V_n \approx 1$ nV/$\sqrt{}$Hz (>100 Hz) was used as preamplifier; its noise contribution can be expressed as $\delta\Phi_{preamp} \approx V_n/(\partial V/\partial\Phi)$.[12] When increasing β_c from 0.1 up to 17, $\partial V/\partial\Phi$ of SQUID increased from 30 μV/Φ_0 up to 600 μV/Φ_0, thus reducing $\delta\Phi_{preamp} \approx V_n/(\partial V\partial\Phi)$ from about 33 to 1.7 $\mu\Phi_0/\sqrt{}$Hz. Table I shows two limiting cases: $\delta\Phi_{preamp}$ dominates $\delta\Phi$ because of a small $\partial V/\partial\Phi$ of SQUID #1 and #2 while $\delta\Phi_s$ dominates $\delta\Phi$ due to a large β_c of SQUID #4. The minimal $\delta\Phi$ of 4.2 $\mu\Phi_0/\sqrt{}$Hz appears at $\beta_c \approx 3.5$ for SQUID #3. Here, both $\delta\Phi_s$ and $\delta\Phi_{preamp}$ contribute to $\delta\Phi$ with the same weight, i.e., $\delta\Phi_s \approx \delta\Phi_{preamp}$.

Voss[9] noted that an excess voltage noise S_v across a tunnel junction is produced by random switching between the superconducting and nonzero voltage states in the case of $\beta_c > 0$. We interpret the increase of $\delta\Phi_s$ up to 7 $\mu\Phi_0/\sqrt{}$Hz by an increase in S_v of the junction with β_c. Indeed, both the Johnson noise and the random switching noise contribute to $\delta\Phi_s$, i.e., $\delta\Phi_s{}^2 = 2k_BT[L_s{}^2/R_J + 2R_d/(\partial V/\partial\Phi)^2] + S_v/(\partial V/\partial\Phi)$. Here, the Johnson noise contains two parts,[15] a static term $2k_BTL_s{}^2/R_J$ and a dynamic term $4k_BTR_d/(\partial V/\partial\Phi)^2$. At $\beta_c \gg 1$, S_v dominates $\delta\Phi_s$.

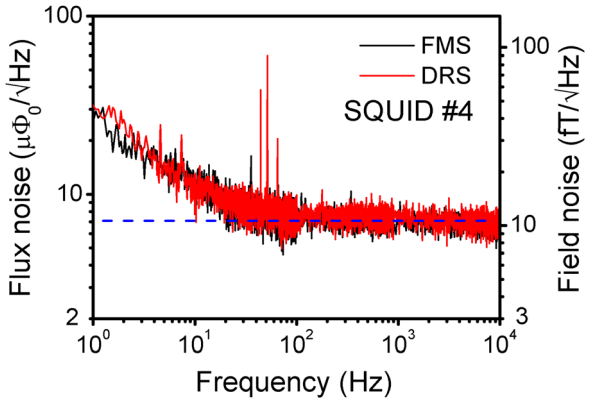

FIG. 2. Noise measurements of SQUID #4 with both DRS and FMS.

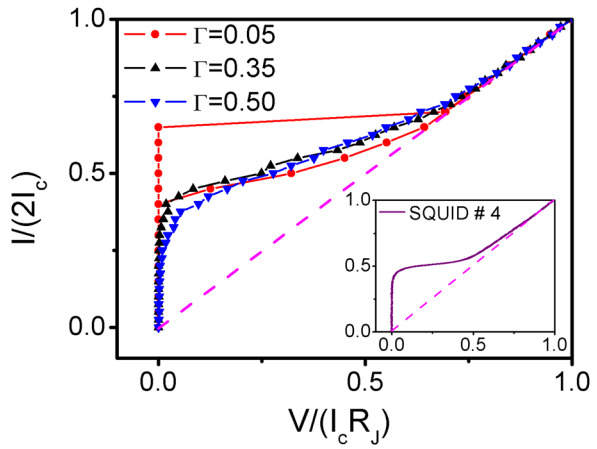

FIG. 3. Three simulated *I-V* characteristics of a symmetric SQUID with $\Gamma = 0.05$, 0.35, 0.5 and $\beta_c = 17$, $\beta_L = 1$, at applied integer flux $\Phi = n\Phi_0$. The inset shows the measured *I-V* characteristic of SQUID #4, which agrees best with the simulation at $\Gamma = 0.35$. Symbols *V* and *I* denote the time averaged dc voltage and bias current.

We introduce an effective noise temperature *T**, which is usually used to characterize amplifier noise, to rewrite the expression above: $\delta\Phi_s^2 = 2k_B \quad (T + T^*)[L_s^2/R_J + 2R_d/(\partial V/\partial\Phi)^2]$. The value of $(T + T^*) \approx 30$ K is obtained for $\beta_c \approx 17$ (SQUID # 4). A $(T + T^*)$ of 30 K leads to the effective thermal rounding parameter, Γ*, of 0.35, which is much larger than the value of $\Gamma = 0.05$ ($T \approx 4.2$ K, $I_c = 4\,\mu$A) used in the simulations by Voss.[9] This high Γ* results in rounding of *I-V* characteristics thus causing the hysteresis to disappear. Figure 3 shows three simulated *I-V* characteristics with different Γ based on the model of a symmetric SQUID[8] with $\beta_c = 17$ and $\beta_L = 2L_sI_c/\Phi_0 \approx 1$. For $\Gamma = 0.05$, the *I-V* characteristic is obviously hysteretic. However, the hysteresis disappears at $\Gamma = 0.35$ ($T = 30$ K), which agrees with the measured *I–V* curve of SQUID #4 shown in the inset. At $\Gamma = 0.5$, the rounding effect is even stronger.

To conclude, the *V*-Φ curves of dc SQUIDs can be generally described as quasi-sine waves when $\beta_c < 1$ and quasi-square waves, if $\beta_c > 1$. Indeed, three important SQUID parameters, the intrinsic noise $\delta\Phi_s$, the voltage-to-flux transfer coefficient $\partial V/\partial\Phi$, and the dynamic resistance R_d, increase with increasing β_c. In our measurements, a hysteresis was not observed up to nominal $\beta_c \approx 17$. The absence of any hysteresis is explained by the large random switching noise, S_v, leading to high values of the effective thermal rounding parameter Γ* and thus, a rounding of the SQUID *I-V* curve. Our simulations support this interpretation.

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